Prime Factorisation of a Number vs. Prime Factorisation of Its Square

Theorem :1.2

If p divides $a^{2,}$ then p divides a, where a is a positive integer.

Proof:

Key Idea:

When you square a number, the prime factors stay the same, but their exponents double.

General Rule:

If a number “a” has the prime factorization, as below.

$a = p1^{\{k1\}} \text{ x } p2^{\{k2\}} \text{ x } p3^{\{k3\}} \text{ x } \ldots\ldots\ldots\ldots \text{ x } pn^{\{kn\}}$

Then its square a^2 will have the prime factorization, as below.

$a^2 = \{p1^{\{k1\}}\}^2 \text{ x } \{p2^{\{k2\}}\}^2 \text{ x } \{p3^{\{k3\}}\}^2 \text{ x } \ldots\ldots\ldots\ldots \text{ x } \{pn^{\{kn\}}\}^2$

1. Example 1:

- Let a = 12.
- Prime factors of 12 are $2^2 \text{ x } 3^1$.
- Square of 12 = 12^2 = 144.
- Prime factors of 144 are $2^4 \text{ x } 3^2$.

2. Example 2:

- Let a = 30
- Prime factors of 30 are $2^1 \text{ x } 3^1 \text{ x } 5^1$.
- Square of 30 = 30^2 = 900.
- Prime factors of 900 are $2^2 \text{ x } 3^2 \text{ x } 5^2$.

3. Example 3:

- Let a = 7 (a prime number).

- Prime factors of 7 is 7^1

- Square of 7 = 7^2 = 49.

- Prime factors of 49 is 7^2.

Visual Summary:

Number "a"	Prime Factors of "a"	a^2	Prime Factors of a^2
6	$2^1 \times 3^1$	36	$2^2 \times 3^2$
10	$2^1 \times 5^1$	100	$2^2 \times 5^2$
15	$3^1 \times 5^1$	225	$3^2 \times 5^2$

Conclusion:

- Squaring a number does not introduce new primes.

- It only doubles the exponents in its prime factorization.